Motion of rotation under variable angular acceleration

It is not necessary that in circular motion the body should move with uniform angular acceleration. Sometimes the circular motion taking place will be under variable acceleration.

we have, $\omega = \frac{d\theta}{dt}$

$$\alpha = \frac{d\omega}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right)$$

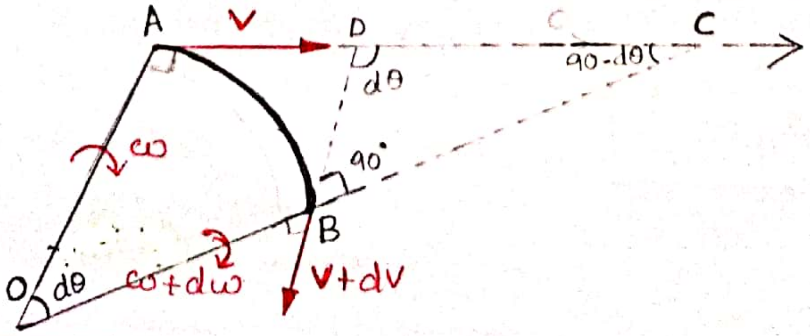
$$\boxed{\alpha = \frac{d^2\theta}{dt^2}}$$

$$\alpha = \frac{d\omega}{dt} \times \frac{d\theta}{d\theta} = \frac{d\theta}{dt} \times \frac{d\omega}{d\theta} = \omega \frac{d\omega}{d\theta}.$$

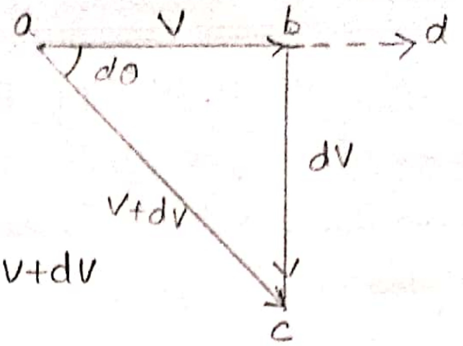
Tangential acceleration & Normal acceleration

When a particle is having curvilinear motion, the total acceleration of the particle is having two components. One component of acceleration along the tangent and the other component is ~~nor~~ normal to the tangent. The component along the tangent is called tangential acceleration (a_t) and the component normal to the tangent is called normal acceleration (a_n)

Expression for tangential & normal acceleration



In vector diagram;



$\vec{ab} \Rightarrow$ represents the velocity vector V

$\vec{ac} \Rightarrow$ represents the velocity vector V+dV

$\vec{bc} \Rightarrow$ represents the change in velocity dV.

Let the component of this change in velocity along the tangential and normal directions are;

Tangential component of change in velocity

$$= (V+dV)\cos d\theta - V \quad [\text{ie, } \vec{bd}]$$

when dθ is very small, then $\cos d\theta \approx 1$

$\therefore$ Then it becomes $V+dV-V = \underline{\underline{dV}}$

Tangential component of acceleration, $a_t = \frac{dV}{dt}$

ie, $a_t = \frac{dV}{dt} = \frac{d\,r\omega}{dt} = r\alpha$

$$\therefore \boxed{a_t = r\alpha}$$

Normal component of change in velocity

$$= (V+dV)\sin d\theta$$

when dθ is very small, $\sin d\theta \approx d\theta$.

$\therefore$ it becomes $(V+dV)d\theta = Vd\theta + dVd\theta$

$dVd\theta$ is small; so neglect it.

$\therefore$ it becomes $Vd\theta$.

Normal component of acceleration, $a_n = \frac{Vd\theta}{dt} = V\omega$ $\quad (\because V = r\omega)$

$$= V \times \frac{V}{r} = \frac{V^2}{r}$$

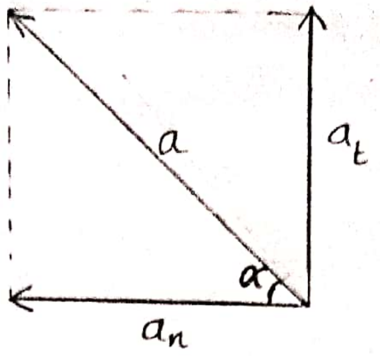
$$\therefore \boxed{a_n = \frac{V^2}{r}}$$

Total acceleration (m/s^2)

We have;

$a_t \rightarrow$ tangential acceleration

$a_n \rightarrow$ normal acceleration



Total acceleration,

$$\boxed{a = \sqrt{a_t^2 + a_n^2}}$$

$$\tan\alpha = \frac{a_t}{a_n}$$

$$\therefore \boxed{\alpha = \tan^{-1}\left(\frac{a_t}{a_n}\right)}$$

The tangential component of acceleration is due to change in magnitude of velocity and normal component is due to change in direction of velocity.

Notes:-

① If the direction of motion does not change, then $a_n = 0$. The direction of motion will not change if it is along a straight line. For the displacement along a straight path, the radius of the circular path is infinitely great and hence the normal component ($a_n = V^2/r$) will be zero. There will be only tangential component of acceleration. i.e, $a_t = \frac{dv}{dt}$

② For the displacement along a circular path, with constant speed, the tangential component of acceleration (i.e, $a_t = \frac{dv}{dt}$) will be zero. Tangential component of acceleration is due to change of magnitude of velocity. For constant speed, $dv = 0$. There will be only normal component of acceleration, $a_n = V^2/r$

③ The direction of the normal component of acceleration will be normal to the velocity and the displacement. As the velocity & displacement are normal to the radius of the circular path, hence the direction of normal component of acceleration will be along the radius towards centre.

(4) The total acceleration of a particle in curvilinear motion is due to change of magnitude of velocity or due to change of direction of velocity or due to both. The change of magnitude of velocity is due to tangential acceleration alone whereas the change of direction of velocity is due to normal acceleration alone.

(8)